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Aiding and opposing mixed-convection heat transfer in a vertical tube: loss of boundary condition at different Grashof numbers

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Mixed-convection heat transfer in a vertical tube with aiding and opposing flows (upflow and downflow heating) was studied experimentally for Reynolds numbers ranging from about 1000 to 30,000 at constant Grashof number under constant wall temperature (CWT) conditions. The Grashof number was varied independently by adjusting steam pressure in the jacket of a vertical, double-pipe heat exchanger. The data corroborate earlier studies that found the way in which Nu is reduced is not identical for aiding and opposing flows. Loss of boundary condition significantly affects normal trends in the data at low Reynolds numbers and is caused by a temperature profile in the wall interacting with low driving force temperature difference.

Keywords: vertical mixed convection; heat transfer enhancement; Grashof number variation

Introduction

This paper is an extension of recent work (Joye 1996a) in which upflow and downflow heat transfer results at the same Grashof number were compared for flow in a vertical tube. In the present study, we attempt to provide the same kind of data at higher and at lower Grashof numbers than in the previous study. In extending the experiments to their limits, the consequences of loss of boundary condition became apparent.

When natural and forced-convection heat transfer mechanisms interact, combined- or mixed-convection is said to exist. The buoyancy force is a gravity effect and always acts vertically, but the forced flow direction is arbitrary. The interactions of natural and forced-convection currents can be very complex, and each case of geometry and orientation must be treated separately. In vertical, internal flows, the forced flow may be upwards or downwards, and the heat transfer may be to or from the fluid in the conduit. Heating in upflow is termed "aiding" flow, because the natural convection currents are in the same direction as the forced flow. Downflow heating is termed "opposing" flow, because the buoyancy currents are opposite. An extensive review paper by Jackson et al. (1989) summarizes much of the work in this field.

Vertical internal flow heat transfer has application to chemical process heat transfer, nuclear reactors, and some aspects of electronic cooling. Significant heat transfer enhancement may be realized by mixed convection in these situations.

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In experimental studies of mixed convection in tubes, the uniform heat flux (UHF) boundary condition at the wall is most often encountered. The other type of boundary condition is constant wall temperature (CWT), which is used in the experiments described in this work. Both boundary conditions have practical importance, but CWT is generally more common in the process industries, particularly when phase change heat transfer —for example, condensing vapors or boiling liquids—occurs in an exchanger. It is also a common practice to treat the wall as a boundary of constant temperature at some average, when the temperature varies along the wall. The UHF boundary condition occurs when electrical or nuclear heating is used, but it also occurs in process situations when the driving force temperature difference between two fluids in an exchanger is identical at each end, as long as average fluid properties and average heat transfer coefficient are used.

Three regions characterize heat transfer in vertical, mixed-convection with CWT boundary condition; whereas, the UHF condition shows only two (Joye 1996a; Joye et al. 1989). A turbulent flow region occurring at high Reynolds numbers shows results independent of upflow or downflow, heating or cooling, CWT or UHF boundary condition. Experimental results in this region agree well with forced-flow correlations; e.g., the Sieder–Tate or any of the newer relationships (Holman 1990). Mixed-convection is not important here, because hydrodynamic turbulence generated at these Reynolds numbers is much stronger than the natural convection mechanism and dominates the heat transfer. For our data, this region occurs above about Re = 10,000, but this is Grashof number-dependent.

At the other extreme, the asymptote region occurs at low Reynolds numbers (about 4000 and below) in CWT situations. This region results from the flow being slow enough so that the

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outlet temperature approaches the wall temperature. This situation has been described previously by Martinelli et al. (1942) and McAdams (1954).

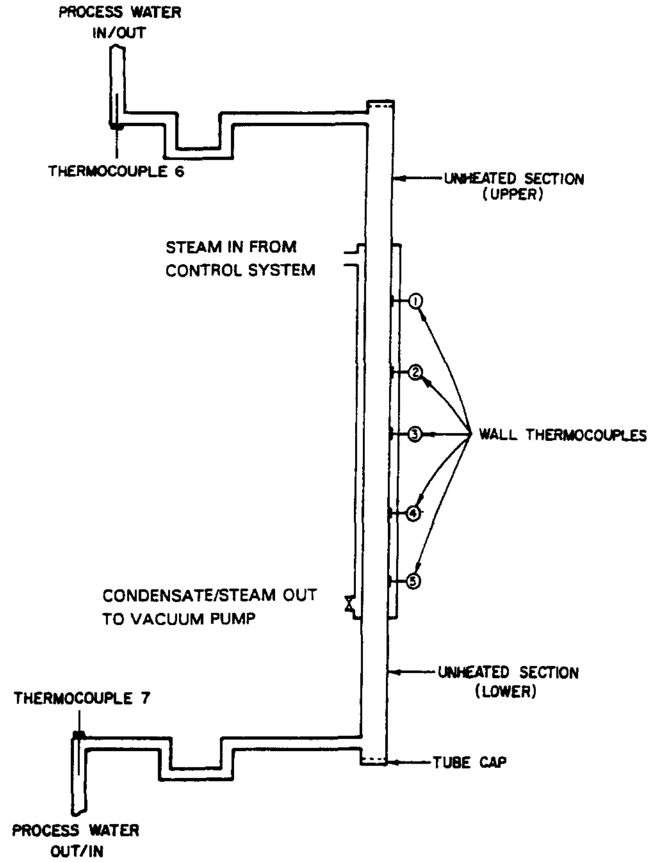
At Reynolds numbers between about 4000 and the upper Reynolds number transition point, the mixed-convection region exists, where both natural convection and forced-convection mechanisms are the same order of magnitude and interact in complex ways. Heat transfer results for upflow heating are significantly different from those for downflow heating in the mixed-convection region and each is commonly studied separately.

Experimental method

The apparatus employed for the present study used steam condensing in the annulus of a copper–copper, double-pipe heat exchanger as the constant wall temperature heat source. Because the condensing film thickens as it flows downwards, the wall temperature is always a bit hotter at the top than at the bottom. As flow rate of liquid increases, this profile becomes more prominent. Thus, the "constant" wall temperature is an average that is quite constant at low flow rates but less so at high flow rates. Steam pressure was controlled by a Nash vacuum pump and a control system comprising several valves. Figure 1 shows the experimental apparatus, which is essentially identical to that used in previous studies by the same author (Joye 1996a; Joye et al. 1989). A vacuum gauge measured the (vacuum) pressure in the jacket, and a pressure gauge measured positive steam pressure when that was used. Five thermocouples were used to measure wall temperature of the inside tube. The vacuum pressures in the jacket ranged from 99.6 to 30.2 kPa absolute pressure (0.5–21 in Hg vacuum, respectively). A run at 345 kPa (35 psig) steam pressure was included to extend the range of the present data.

Steam pressure could be controlled by manipulating valves in the control system. Constant wall temperature was maintained by manipulation of the control system. At low pressures, the steam sometimes condensed at the inlet only, and when this happened, the steam valves were adjusted to provide sufficient steam flow so that reasonably constant wall temperature was evident by readings on the digital thermometer. Loss of steam flow also affected the heat transfer results; it gave a significant deviation from the expected Nusselt number. Wall temperatures and, hence, Grashof numbers could be altered by adjusting the steam pressure within the limits of the equipment. Numerous repeat runs on individual points in question due to this experimental difficulty were carried out. Once the experimental conditions were set correctly, repeatability was within $\pm 10\%$.

Heated inlet water was used in this investigation in an effort to get lower Grashof numbers by reducing the approach temperature difference between steam and inlet water. This succeeded only partially, because the increased temperatures altered the fluid properties (especially the viscosity), which tended to increase the Grashof number. However, some extension of the range of data was accomplished.



NOTE: PIPES ARE INSULATED BETWEEN THERMOCOUPLES 6 AND 7.

Figure 1 Experimental apparatus; test section detail

Copper–constantan thermocouples were used as before to measure both wall temperature and inlet/outlet temperatures. The unit was insulated to minimize heat loss, particularly important at the outlet. Mixing elbows were used to get a well-mixed outlet temperature in both upflow and downflow. Two rotameters were used to measure the flow rates at high and at low

Notation

C_p	fluid heat capacity, kJ/(kg-K)
D	tube diameter, m
g	gravitational acceleration, m/s^2
Gr_{bD}	Grashof number based on properties evaluated at the bulk average fluid temperature and tube diameter, $\rho^2 g D^3 \beta \Delta T/\mu^2$
Gz	Graetz number, wC_p/kL
h	film heat transfer coefficient, $W/(m^2\text{-}K)$
k	fluid thermal conductivity, W/(m-K)
L	heated length of tube, m
Nu	Nusselt number, hD/k
Pr	Prandtl number, $\mu C_p/k$
Re	Reynolds number, $Dv\rho/\mu$
T	temperature, K
ΔT	temperature difference (K) average wall to average bulk fluid for Grashof number, unless otherwise noted
w	mass flow rate, kg/s
v	average fluid velocity, m/s

Greek

β	volume expansivity, 1/K
μ	viscosity, Pa·s
ρ	density, kg/m^3

throughput. The L/D of the heated section was 49.6; the inside diameter of the central tube was .032 m (1.265 in.). There was an unheated length of about 0.25 m (10 in.) before the entrance of the heated section. Water in the tube was prevented from boiling or degassing by pump pressures of 200–400 kPa gauge (30–60 psig). About 100 kPa gauge is required to avoid degassing.

Heat transfer data, including all dimensionless groups, coefficients, etc., were calculated on spreadsheet software. Heat transfer coefficients (film coefficients) were calculated from Newton's Law of Cooling, where the heat rate (W or Btu/hr) was calculated from the temperature rise and the flow rate of water, and an arithmetic average temperature driving force was used. (The log-mean temperature driving force was also used for comparison). All the dimensionless groups (Nusselt number, Prandtl number, Reynolds number, and Grashof number) were based on the arithmetic average (bulk fluid) temperature, although some prefer other temperature bases (Jackson et al. 1989) for Grashof number. Conversion to other bases can be done, if all the appropriate temperatures are known. However, the different bases show very similar trends as Grashof number is changed, so the temperature basis itself is not so important, fundamentally, but the particular definition must be known for the sake of computation and comparison.

Results and discussion

Water was used as the test fluid, and the Prandtl number (an uncontrolled parameter) did not vary much from about 2.8, because of the high water temperatures due to the heated inlet water. The Grashof numbers based on diameter and bulk fluid properties were constant for each run to within $\pm 20\%$, except at the highest steam pressure, where the variation was about $\pm 36\%$. The upflow/downflow experiments were performed at different times, and the respective Grashof numbers differed by about 8%, so the Grashof numbers for corresponding upflow and downflow conditions are essentially identical. The corresponding Grashof number based on length can be computed easily, because L/D = 49.6 for all cases. This is sufficiently long so that entrance effects are negligible.

Earlier investigators have found that, if L/D is lower than about 30, entrance effects become significant; if L/D is lower than about 10, entrance effects dominate. Because we use only one L/D here, and it is in the region where entrance effects are negligible, we presume the results of average heat transfer coefficient would apply to all L/D over 30.

Asymptotic limits of the data

Mixed-convection data are usually compared to the forced-convection case via a turbulent, forced-flow only correlation. From this, increases in heat transfer attributable to natural convection effects can be followed clearly. In this work, we prefer the Sieder–Tate equation with viscosity correction as the turbulent flow base case,

$$\mathrm{Nu}/\mathrm{Pr}^{1/3}\phi_v^{0.14} = .023\ \mathrm{Re}^{0.8} \tag{1}$$

which is often referred to in short-hand as the ".023" equation. For heating situations, the viscosity correction factor ϕ_v is often folded into a slightly different power for Pr than above; for example, the correlation of Swanson and Catton (1987) for opposing flow shows 0.5 power of Prandtl number. The Jackson and Fewster (Jackson et al. 1989) correlation uses the Petukhov–Kirillov relationship for the base case, which gives numerical results only slightly different from the Sieder–Tate. In any event, these base cases are all reasonably comparable.

Martinelli et al. (1942) and McAdams (1954) both show an asymptotic relationship for CWT conditions at low Reynolds numbers. This is a result of losing ΔT driving force at the exit and, consequently, would not be expected to hold in experiments with UHF at the wall (this maintains ΔT driving force). The equation takes various forms, but fundamentally it is as follows:

$$\mathrm{Nu}_{\mathrm{asymptote}} = (2/\pi)\ \mathrm{Gz} \tag{2}$$

where Gz is the well known Graetz number. When Re is used as an independent parameter, as we prefer, the asymptote is Pr number = dependent,

$$\mathrm{Nu}_{\mathrm{asymptote}} = 0.5\ \mathrm{Re}\,\mathrm{Pr}\,D/L \tag{3}$$

which translates to

$$\mathrm{Nu}_{\mathrm{asymptote}}/\mathrm{Pr}^{1/3}\phi_v^{0.14} = 0.5\ \mathrm{Re}\,\mathrm{Pr}^{2/3}D/L \tag{4}$$

in the equivalent Sieder–Tate style formulation.

Experimental results

The upflow heating results (Nusselt number as a function of Reynolds number) were very similar to results previously obtained (Joye 1996a). As the Grashof number is increased, the enhancement in heat transfer due to mixed convection increases, as expected. We were not able to get uniform heating of the tube at steam pressures less than about 30 kPa absolute; however, the lowest approach temperature we were able to achieve was about 2°C, one-twentieth of that previously. At this level, measurement errors in temperature resulted in a large relative error in Nusselt number, which made that data unsuitable for reporting here. At large approach temperatures (higher Gr), the temperature measurement error is very small compared to ΔT, and relative error in Nusselt number due to temperature measurement is insignificant (Joye 1996a). At high flows (high Re) the same thing happens with respect to the driving force ΔT, but the opposite happens with respect to temperature rise ΔT. From repeat runs, we estimate the relative error in Nu to be about $\pm 10\%$ at Re greater than about 7000. This estimate applies regardless of Re at the higher Gr. However, at the lower Gr, we estimate the relative error in Nusselt number to be about $\pm 25\%$ on the same basis.

Upflow/downflow comparison

Figures 2–4 show the comparison between an individual upflow heating run and its equivalent downflow heating conditions at the same Grashof number for different Grashof numbers. The Grashof number based on bulk average temperature is used in these figures instead of the Grashof number based on film temperature. This puts all dimensionless groups on the same temperature basis, which many investigators prefer. The two Grashof numbers are related by $\mathrm{Gr}_{bD} \approx 0.32\,\mathrm{Gr}_{fD} \pm 25\%$ in our experiments (this is not a general relationship; it is very temperature-dependent and changes throughout the flow range). The same trends as previously reported (Joye 1995) seem to exist. The aiding flow results seem to be shifted both vertically and horizontally as Gr is reduced, but the opposing flow data seem to be shifted vertically only.

By now, several peculiarities in the upflow heating case are well known (Poskas 1994; Petukhov and Nolde 1959; Zeldin and Schmidt 1972; Connor and Carr 1978). There is a "dip" below the forced flow line in all cases, sometimes referred to as "de-enhancement" or "buoyancy-retarded" heat transfer. These regions have been characterized quantitatively by various investigators (Joye 1996a; Jackson et al. 1989; Poskas et al. 1994;

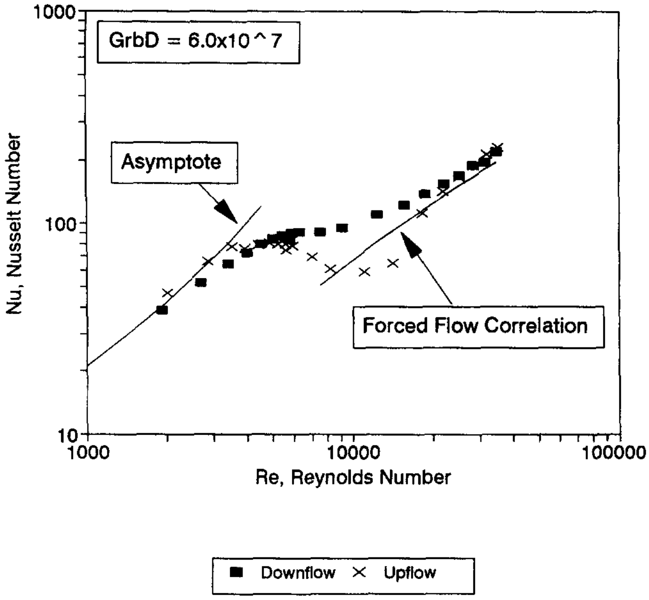


Figure 2 Upflow and downflow heating data compared at same Grashof number; Shell-side steam pressure = 345 kPa abs. ($Gr_{bD} = 6 \times 10^7$)

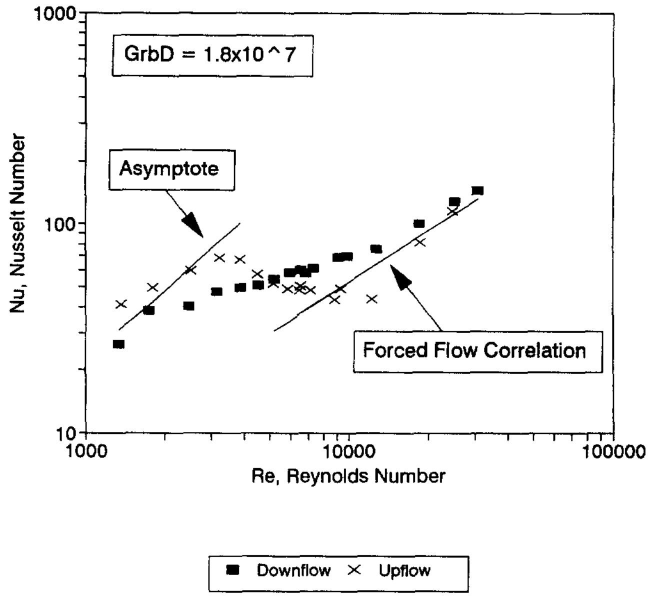


Figure 3 Upflow and downflow heating data compared at same Grashof number; Shell-side steam pressure = 99.6 kPa abs. ($Gr_{bD} = 1.8 \times 10^7$)

Connor and Carr 1978) using theories of Hall and Jackson in Jackson et al. (1989) as a guide. There is still much debate as to the best formulation, however.

Figures 2–4 also show the crossover phenomenon between the upflow data and the downflow data, as previously reported (Joye 1996a). This persists to higher and lower Grashof numbers than before, and the trend seems to be consistent. At the higher Grashof numbers, the downflow heating data are above the upflow heating data, but as Gr is reduced, the downflow heating data seem to be more strongly influenced by Grashof number change than the upflow heating data, and the crossover is produced. The downflow data soon become partly above and partly below the upflow data.

A satisfactory explanation of this phenomenon is presently lacking, but the effect may be a result of laminarization of the flow for aiding flow conditions, (Scheele and Hanratty 1963; Scheele et al. 1960; Hanratty et al. 1958) so that the data follow the asymptote further into the Reynolds number range (higher flow rates) than the equivalent downflow situation. It may also be caused by a backflow in the inlet region (Joye and Jacobs 1994) in the downflow case, which disappears at lower Grashof number with the same Reynolds number. Backflow turbulence would enhance heat transfer. It is possible that loss of boundary condition may also have an effect.

Loss of boundary condition

Figures 5 and 6 show what happens to the data as boundary condition becomes less sharp. Experimental detail is given in Wojnovich (1995). The wall temperature was never exactly constant in these experiments, but was hotter at the top where steam enters and cooler at the bottom, due to a thicker condensate film on the steam side. When the approach temperatures are low, the driving force is low, and the temperature profile becomes significant, but it seems to do so only in the low Reynolds number, "relaminarized" region. The effect for upflow is to shift the data upwards from the asymptote line; the effect in downflow is similar but in the opposite direction. We believe the effect has to do with the temperature profiles between wall and fluid. The wall profile has always the same kind of slope, hotter at the top and cooler at the bottom. In downflow heating, the fluid temperature is cooler at the top and hotter at the bottom, giving a driving force profile similar to a co-current, double-pipe heat exchanger. In upflow, this situation is reversed, giving a temperature profile typical of a countercurrent, double-pipe heat exchanger. Figures 7 and 8 illustrate these effects. It is possible in the upflow case for the outlet temperature to be higher than the computed average wall temperature. In fact, this occurred in our data, and the effect was to move the data off the asymptote, as shown in Figures 5 and 6. Lower Grashof numbers in these experiments were obtained by lowering the steam pressure and, thereby, lowering the wall temperature. As this progressively occurred, the consequence of loss of boundary condition became apparent.

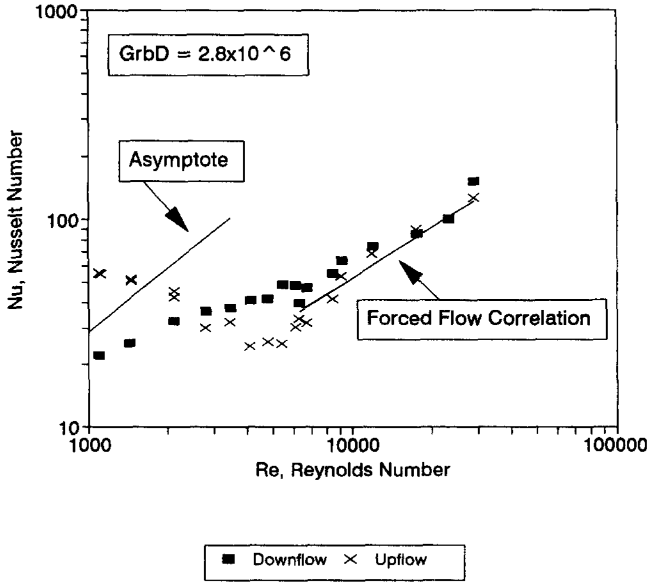


Figure 4 Upflow and downflow heating data compared at same Grashof number; Shell-side steam pressure = 67.4 kPa abs. ($Gr_{bD} = 2.8 \times 10^6$)

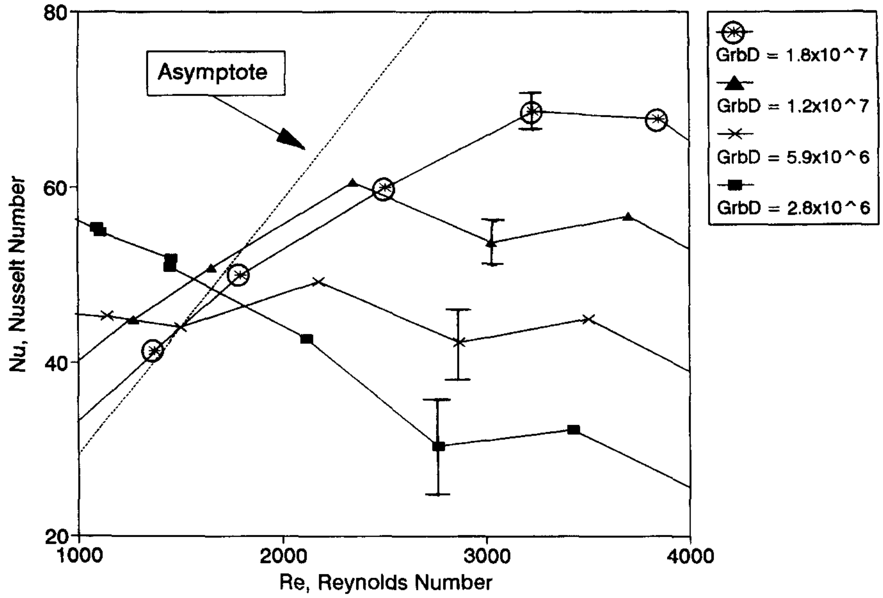


Figure 5 Loss of CWT boundary condition at low Reynolds number causes positive deviation from asymptote for aiding flow; estimate of error shown by bars

The deviation from the asymptote becomes more marked as Grashof number decreases. The effect is quite prominent for $Gr_{bD} = 2.8 \times 10^6$.

In the downflow case, the outlet temperature is suppressed, as it were, from reaching what it should be under exactly constant wall temperature conditions, because of the co-current profile in the wall temperature. The deviation from the asymptote at low Reynolds numbers in our data for downflow was not the result of exit cooling, as we initially suspected. As a check, we temporarily increased flow once steady state had been achieved and recorded the hottest outlet temperature. This would correspond to the true outlet temperature in the absence of exit line cooling. The data in Figures 2–4 were computed with this "procedure" for the lowest two Reynolds numbers. At higher Re, there was no exit line cooling in the data (we got no change in the maximum outlet temperature when we performed the procedure).

Thus, the boundary condition will have a very significant effect on the data at low Reynolds numbers. It is possible to have the same deviation from the asymptote in downflow heating as occurred in Figure 5 for upflow, if the temperature profile is more similar to countercurrent two-fluid exchange (Figure 7). Because of the temperature profile, the deviation from the asymptote could be above rather than below, giving data more similar to the UHF boundary condition case, which does not show an asymptote at all.

At higher Reynolds numbers, by contrast, there seems to be no effect of boundary condition in the data at all. This includes CWT, UHF (correlations developed for UHF work well for CWT

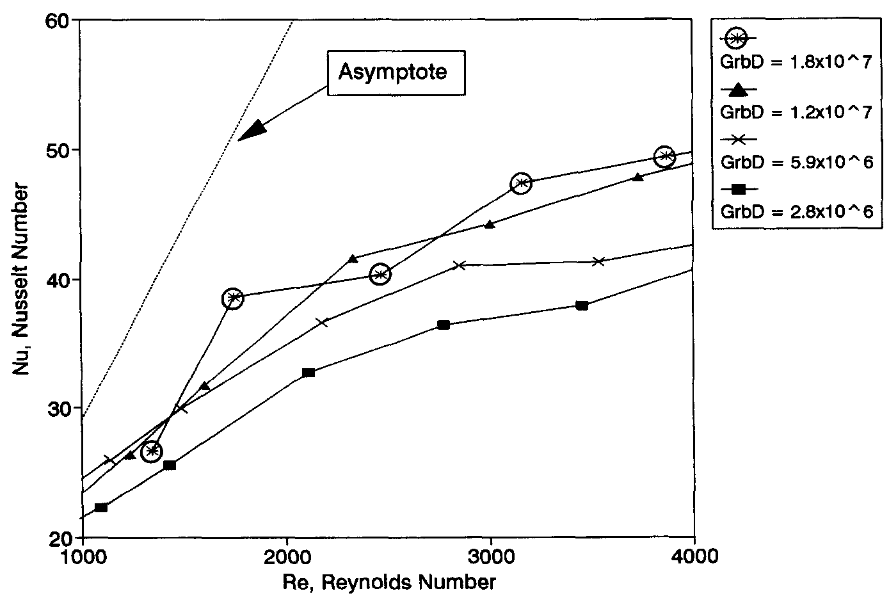


Figure 6 Loss of CWT boundary condition at low Reynolds number causes negative deviation from asymptote for opposing flow; same estimate of error as in Figure 5

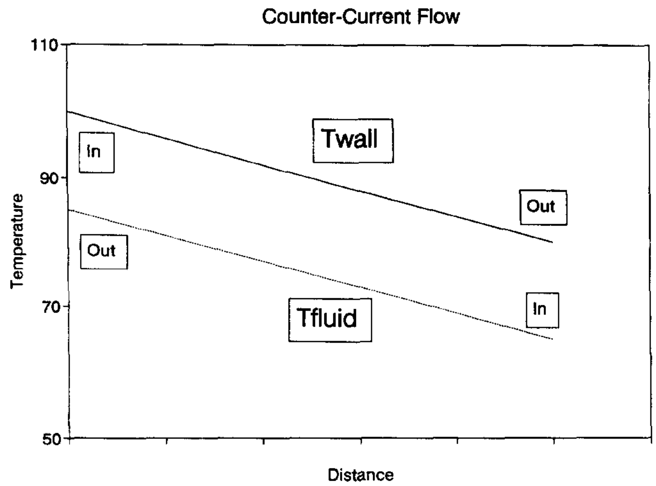


Figure 7 Counter current temperature profiles in upflow and downflow heating with nonconstant wall temperature

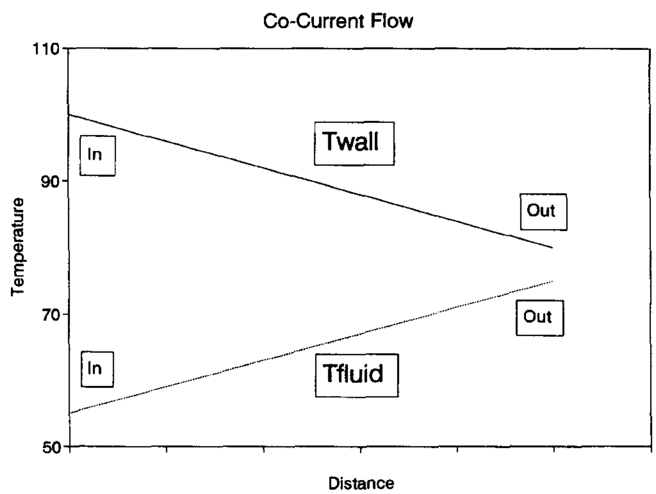


Figure 8 Co-current temperature profiles in upflow and downflow heating with nonconstant wall temperature

also in this region (Joye 1996b), constancy of wall temperature or anything else. Low approach temperatures merely increased the scatter in the data (Wojnovich 1995) rather than creating any deviations from expected results. To the authors' knowledge there have been no similar studies on effects of boundary condition on vertical, internal, and mixed-convection heat transfer.

Conclusions

Mixed-convection flows in a vertical tube show significant heat transfer enhancement over the forced-flow only case, whether the flow is aiding or opposing. Opposing flow generally gives higher heat transfer enhancement than aiding flow in the mixed-convection region. However, as Gr is reduced in this region, a crossover phenomenon occurs, where the above is true for only a part of the Reynolds number range.

Loss of boundary condition significantly affects heat transfer results only at low Reynolds numbers (laminar regime), where it increases Nusselt number for aiding flow and decreases Nusselt number for opposing flow, relative to the asymptote condition for heat transfer under a constant wall temperature boundary condition. This effect is tied to the temperature profiles in wall and fluid.

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